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Publication date:
1990

Document Version
Publisher's PDF, also known as Version of record

CentER for Economic Research

REPRINT

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Reprinted from Contributions to Operations Research and Economics, Cambridge: MIT Press, 1989

Reprint Series
no. 33

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ISSN 0924-7874

CentER REPRINT
for
Economic Research

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13 Toward a Levels Version of the Rotterdam and Related Demand Systems

Anton P. Barten

13.1 Introduction

The theory of demand for the individual consumer implies a set of properties (constraints) on the elasticities of demand with respect to income (or total expenditure) and prices. For at least two reasons it is desirable to take these properties into account in empirical work: The first is the reduction in the number of independent coefficients to be estimated. The second is the ability to obtain predictions with estimated versions of the demand relations that make sense from a theoretical point of view. This last possibility is attractive also if one works with data for the whole economy rather than for a single consumer. Indeed, without the fiction of the representative consumer, it is difficult to give any meaning to empirical results for an aggregate of consumers.

Besides the homogeneity condition the constraints on the elasticities pertain to more than one demand function at a time. To take the constraints into account in a proper way, one has to formulate and estimate a complete system of demand equations, which in principle describes how the consumer allocates his budget over all desirable goods and services.

The Theil (1965) formulation of what is known as the Rotterdam demand system amounts to a convenient and simple transformation of demand elasticities into constants that satisfy, or can be made to satisfy, the theoretical constraints. They can be directly estimated. Of course, the Rotterdam system is not the only demand system that (1) can or does incorporate constraints from theory, (2) is relatively easy to estimate and interpret, and (3) is potentially flexible (i.e., allows for nontrivial interactions among commodities, such as specific substitution or complementarity). Still, the Rotterdam system is not a priori dominated by any other system, and it is therefore useful to increase its applicability.

Indeed, as originally formulated, the Rotterdam specification applies to a system in terms of the logarithmic first differences in quantities, prices, and incomes. This limits its practical use to the analysis of time series data. Even for that type of data more refined dynamics are difficult to capture using first differences of the major determinants. Moreover there are cross sections of observations with (sometimes imputed) price variation, which can only be meaningfully handled by a system in terms of the levels of the variables. Such a system with a Rotterdam-type parametrization appears to be a useful tool for demand analysis. It is the purpose of this chapter to present such a system.

The first-difference version of the Rotterdam system is one of a class of systems to which the almost ideal demand system (AIDS) of Deaton and Muellbauer (1980)—in first differences—and the CBS demand system of Keller and Van Driel (1985) also belong. For the levels version of the Rotterdam system, a similar class can be formulated. The counterpart of the AIDS first-difference equation in this class is not quite the same as the levels version of AIDS proposed by Deaton and Muellbauer, although its parametrization is the same.

It is useful to start with a presentation of the constraints on the elasticities which are derived from demand theory. This is the topic of section 13.2. Section 13.3 takes up the case of convenient parametrization in systems in terms of first differences. We then turn to a discussion of the choice of levels versions for these systems. Alternative approaches are also considered. Such systems are used to generate information about quantities demanded, expenditure shares, and the like. For some of these systems, such simulation is not trivial, as is shown in section 13.6. Some insight about the relative merits of the various systems can be gained from an empirical application, which one finds in section 13.7. The last section is devoted to concluding remarks.

13.2 Constraints on Elasticities

As a starting point we use the double-logarithmic demand function

$$\ln q_i = \alpha_i + \eta_i \ln m + \sum_j \mu_{ij} \ln p_j, \qquad i, j = 1, \ldots, n, \tag{1}$$

where q_i is the (positive) quantity of good i and p_i its (positive) price, and m is total expenditure defined as

$$m = \sum_j p_j q_j. \tag{2}$$

The η_i are income or expenditure elasticities; the μ_{ij} are the price elasticities. There is no fundamental reason why these elasticities are constant (i.e., independent of m and the p_j). The same is true for the intercept α_i.

Frisch (1959) states a set of properties that the η_i and the μ_{ij} should satisfy if they are to reflect utility-maximizing behavior. These properties involve the budget shares, w_i, defined as

$$w_i = \frac{p_i q_i}{m}, \tag{3}$$

that is, the share of expenditure on good i in total expenditure. Clearly

$$\sum_i w_i = 1. \tag{4}$$

The first set of properties are those of *adding-up*:

$$\sum_i w_i \eta_i = 1 \qquad \text{(Engel aggregation)}, \tag{5}$$

$$\sum_i w_i \mu_{ij} = -w_j \qquad \text{(Cournot aggregation)}. \tag{6}$$

These properties guarantee that explained demand satisfies the budget identity (2). Next is the *homogeneity* condition:

$$\sum_j \mu_{ij} = -\eta_i, \tag{7}$$

which is derived from the linear homogeneity in m and the p_j of the budget identity (2).

Further properties can be conveniently formulated in terms of the *Slutsky* or compensated price elasticity, defined as

$$\varepsilon_{ij} = \mu_{ij} + \eta_i w_j, \tag{8}$$

which reflects the substitution effect of price changes, with utility kept constant. Note that adding-up conditions (5) and (6) imply an adding-up condition for the Slutsky elasticities,

$$\sum_i w_i \varepsilon_{ij} = \sum_i w_i \mu_{ij} + \sum_i w_i \eta_i w_j = -w_j + w_j = 0 \qquad \text{(Slutsky aggregation)}, \tag{9}$$

while it follows from homogeneity condition (7) and from (4) that

$$\sum_j \varepsilon_{ij} = \sum_j \mu_{ij} + \eta_i \sum_j w_j = -\eta_i + \eta_i = 0 \tag{10}$$

which is the homogeneity condition for the Slutsky elasticities.

An additional property is that of Slutsky *symmetry*:

$$w_i \varepsilon_{ij} = w_j \varepsilon_{ji}. \tag{11}$$

The *negativity* property (not mentioned by Frisch) amounts to

$$\sum_i \sum_j x_i w_i \varepsilon_{ij} x_j < 0, \tag{12}$$

for all x_i that are not constants. These two properties derive from continuity and strong quasi-concavity properties of the utility function.

A further property is not purely theoretical. If the preference ordering can be represented by a utility function that is a sum of n functions $h_i(q_i)$, then

$$\varepsilon_{ij} = \varphi\eta_i(\delta_{ij} - \eta_j w_j), \tag{13}$$

with φ being the reciprocal of what Frisch terms "money flexibility," and δ_{ij} a Kronecker delta. Equation (13) states what is known as the (complete) *want* or *preference independence property*. The linear expenditure system (LES), for instance, is characterized by such independence. Property (13) is attractive in the sense that besides income elasticities η_i, one needs only one other magnitude, φ, to determine all Slutsky elasticities. This extreme reduction in parameters corresponds to an extremely rigid representation of interactions among goods in the preference order. Whether this is acceptable depends on the empirical context.

Apart from the homogeneity property, the constraints mentioned above involve budget shares, which are in principle and in practice variable. The constraints for constant elasticities cannot be applied to variable budget shares. If one is only interested in saving degrees of freedom, one could work with constant elasticities, using a single set of w_i in the constraints. That means, inter alia, that (2) is not respected for the explained q_j except for the sample point for which the selected w_i are valid. It is clearly more desirable to work with a parametrization that allows the use of constraints without impairing the simulation properties of the demand equations.

13.3 Parametrization

The choice of constraints underlying the Rotterdam demand system can be conveniently explained, starting from a double-logarithmic demand function in differential form

$$d\ln q_i = \eta_i \, d\ln m + \sum_j \mu_{ij} \, d\ln p_j, \tag{14}$$

with the η_i and the μ_{ij} being, as before, income and price elasticities, respectively. Note that (14) is not simply (1) in differential form unless η_i and μ_{ij} are constants.

An alternative version of (14) is obtained by using the Slutsky elasticities defined in (8):

$$d\ln q_i = \eta_i(d\ln m - \sum_j w_j d\ln p_j) + \sum_j \varepsilon_{ij} d\ln p_j. \tag{15}$$

In view of (2),

$$d\ln m = \sum_j w_j d\ln q_j + \sum_j w_j d\ln p_j. \tag{16}$$

Writing

$$d\ln Q = \sum_j w_j d\ln q_j, \tag{17a}$$

$$d\ln P = \sum_j w_j d\ln p_j, \tag{17b}$$

we have from (16)

$$d\ln m = d\ln Q + d\ln P. \tag{18}$$

We may then also write (15) as

$$d\ln q_i = \eta_i d\ln Q + \sum_j \varepsilon_{ij} d\ln p_j. \tag{19}$$

The second term in (19) represents the substitution effect of price changes, with utility kept constant. The first term represents the change in demand because of a change in utility. To see this, we make use of the second law of Gossen: $\partial u(q)/\partial q_j = \lambda p_j$, where $u(q)$ is the utility function and λ a (positive) Lagrange multiplier. Then $w_j = (1/\lambda m)\partial u(q)/\partial \ln q_j$, and

$$\begin{aligned} d\ln Q = \sum_j w_j d\ln q_j &= \left(\frac{1}{\lambda m}\right) \sum_j \left(\frac{\partial u(q)}{\partial \ln q_j}\right) d\ln q_j \\ &= \left(\frac{1}{\lambda m}\right) du. \end{aligned} \tag{20}$$

The $d\ln Q$ variable can be seen as the change in the logarithm of real income.

The Rotterdam Specification

By multiplying both sides of (19) by w_i and using

$$b_i = w_i \eta_i, \tag{21}$$

$$s_{ij} = w_i \varepsilon_{ij}, \tag{22}$$

we obtain

$$w_i \, d\ln q_i = b_i \, d\ln Q + \sum_j s_{ij} \, d\ln p_j. \tag{23}$$

Note that the sum over i of the variable on the left-hand side is equal to the log change in real income.

From (5) we have as an adding-up property,

$$\sum_i b_i = 1 \qquad \text{(Engel aggregation)}, \tag{24}$$

while the s_{ij} satisfy

$$\sum_i s_{ij} = 0 \qquad \text{(Slutsky aggregation)}, \tag{25}$$

$$\sum_j s_{ij} = 0 \qquad \text{(homogeneity)}, \tag{26}$$

$$s_{ij} = s_{ji} \qquad \text{(symmetry)}, \tag{27}$$

$$\sum_i \sum_j x_i s_{ij} x_j < 0 \qquad (\text{negativity}, x_i, x_j \neq \text{constant}), \tag{28}$$

$$s_{ij} = \varphi b_i(\delta_{ij} - b_j) \qquad \text{(preference independence)}. \tag{29}$$

All of these constraints are formulated in terms of constants only. The two adding-up conditions (24) and (25) guarantee satisfaction of (17a) for the $d\ln q_i$.

As follows from (21), the b_i represent the (constant) marginal propensities to consume since

$$b_i = w_i \eta_i = \frac{p_i q_i}{m} \frac{\partial \ln q_i}{\partial \ln m} = p_i \frac{\partial q_i}{\partial m} = \frac{\partial (p_i q_i)}{\partial m}. \tag{30}$$

They are also called marginal budget shares in order to distinguish them from the w_i, the (average) budget shares. Constant b_i mean linear Engel curves with convergence of the b_i and the w_i for increasing values of m. Negative b_i, indicating inferior goods, are difficult to reconcile with this type of asymptotic behavior. There are clearly limits to the validity of the Rotterdam specification.

In the transition from differentials to time subscripted finite differences, the w_i on the left-hand side of (23) is replaced by

$$\bar{w}_{it} = \frac{w_{i,t} + w_{i,t-1}}{2}, \tag{31}$$

and a disturbance term (v_{it}) is added. Eventually, we may add an intercept (u_{i0}) and other variables (Δz_{kt}) representing shifts in demand caused by determinants other than income and the prices. The final specification is

$$\bar{w}_{it}\,\Delta\ln q_{it} = b_i\,\Delta\ln Q_t + \sum_j s_{ij}\,\Delta\ln p_{jt} + \Big(a_{i0} + \sum_k a_{ik}\Delta z_{kt}\Big) + v_{it}, \tag{32}$$

with

$$\Delta\ln Q_t = \sum_j \bar{w}_{jt}\,\Delta\ln q_{jt}. \tag{33}$$

Given this definition and adding-up conditions (24) and (25), we have the additional adding-up conditions

$$\sum_i v_{it} = 0, \tag{34}$$

$$\sum_i a_{il} = 0, \qquad l = 0, 1, \ldots. \tag{35}$$

The CBS Specification

Keller and van Driel (1985) propose a specification that treats the s_{ij} of (22) and the

$$c_i = w_i(\eta_i - 1) \tag{36}$$

as constants, but not the b_i. Their version—the CBS version—of (23) reads

$$w_i(d\ln q_i - d\ln Q) = c_i\,d\ln Q + \sum_j s_{ij}\,d\ln p_j. \tag{37}$$

Here the c_i satisfy the adding up condition

$$\sum_i c_i = 0 \tag{38}$$

as can be readily verified.

The dependent variable in (37) is $w_i\,d\ln(q_i/Q)$. Note that the sum of these variables over i equals zero and that $d\ln(q_i/Q)$ basically is the deviation of the relative change in q_i from the average relative quantity change.

As is obvious from (36) and (21) $c_i = b_i - w_i$, with the b_i being the (now variable) marginal propensity to consume and w_i, as before, the average propensity to consume good i. A positive c_i means $b_i > w_i$ or an income elasticity larger than one (i.e., i is a luxury good). A negative c_i means that η_i is smaller than one (i is a necessity). In general, $c_i = \partial w_i/\partial\ln m$. A negative value of c_i implies that for increasing m, w_i turns negative, which is inadmissible. For positive c_i, w_i may become larger than unity, which is also inadmissible. It is clear that the CBS specification (and for the same reason, the AIDS specification) cannot claim global validity, except for the trivial case of $c_i = 0$ for all i.

Another disadvantage of the CBS specification is that preference independence cannot be specified in terms of constants as in the Rotterdam case.

The CBS estimating equation in terms of first differences takes the form

$$\bar{w}_{it}\,\Delta\ln\left(\frac{q_{it}}{Q_t}\right) = c_i\,\Delta\ln Q_t + \sum_j s_{ij}\,\Delta\ln p_{jt} + (a^c_{i0} + \sum_k a^c_{ik}\Delta z_{kt}) + v^c_{it}, \tag{39}$$

with additional adding-up properties similar to (34) and (35). Note that the sum over i of the dependent variables equals zero.

The AIDS Specification

In their development of the almost ideal demand system (AIDS), Deaton and Muellbauer (1980) employ as constants the c_i defined by (36) and the r_{ij}, which are defined as

$$r_{ij} = w_i(\varepsilon_{ij} + \delta_{ij} - w_j) = s_{ij} + w_i\delta_{ij} - w_i w_j. \tag{40}$$

Using this expression to eliminate the s_{ij} from the right-hand side of (37), we obtain with some rearrangement

$$w_i(d\ln q_i - d\ln Q + d\ln p_i - d\ln P) = c_i\,d\ln Q + \sum_j r_{ij}\,d\ln p_j, \tag{41}$$

where $d\ln P$ is defined as in (17b).

The variable on the left-hand side is, in view of (18),

$$w_i(d\ln q_i + d\ln p_i - d\ln m) = w_i\,d\ln w_i, \tag{42}$$

which is the relative change in the expenditure share of good i multiplied by the expenditure share of i itself. Since

$$w_i\,d\ln w_i = w_i\left(\frac{dw_i}{w_i}\right) = dw_i, \tag{43}$$

this variable is simply the change in the expenditure share of good i.

It is easily verified from (40), (4) and from (25) through (27) that the r_{ij} satisfy

$$\sum_i r_{ij} = 0 \qquad \text{(AIDS aggregation)}, \tag{44}$$

$$\sum_j r_{ij} = 0 \qquad \text{(homogeneity)}, \tag{45}$$

$$r_{ij} = r_{ji} \qquad \text{(symmetry).} \tag{46}$$

There is no counterpart of the negativity condition (28) in terms of constant parameters. It is also not possible to specify preference independence in constants only.

It follows from (40) and (22) that the Slutsky elasticities ε_{ij} can be expressed in terms of the r_{ij} and the (variable) w_i, w_j by

$$\varepsilon_{ij} = \left(\frac{r_{ij}}{w_i}\right) - \delta_{ij} + w_j. \tag{47}$$

Transition to finite differences, addition of a disturbance term, and eventually an intercept and additional variables results in the AIDS estimating equation:

$$\Delta w_{it} = c_i \Delta \ln Q_t + \sum_j r_{ij} \Delta \ln p_{jt} + (a_{i0}^A + \sum_k a_{ik}^A \Delta z_{kt}) + v_{it}^A, \tag{48}$$

with the same types of additional adding-up properties as the other two systems. As in the CBS system the dependent variables add up to zero.

A further qualification is in order. In their presentation of AIDS, Deaton and Muellbauer use two alternatives to specify $d\ln P$. The first is consistent with the expenditure function on which their derivation of AIDS is based and involves the r_{ij}. The second is an approximation of that concept and is the same as the one used here, namely, (17b). The main reason for using it here is to have a system which is linear in the unknown coefficients and which has also, as will become clear in the next subsection, the same variables on the right-hand side as in the two other systems.

A Class of Systems

Note that as far as variables are concerned, the right-hand sides of the demand equations of the three systems are basically equal. Since the dependent variables are different, the coefficients on the right-hand side are interpreted differently across these three systems.

A natural extension to a class of systems can be obtained by taking a convex combination of the dependent variables for each system:

$$\begin{aligned}(1 - \theta_2)\bar{w}_{it} \Delta \ln q_{it} - \theta_1 \bar{w}_{it} \Delta \ln Q_t + \theta_2 \Delta w_{it} \\ = d_i \Delta \ln Q_t + \sum_j t_{ij} \Delta \ln p_{jt}(a_{i0}^M + \sum_k a_{ik}^M \Delta z_{kt}) + v_{it}^M,\end{aligned} \tag{49}$$

with $0 \leq \theta_1 \leq 1$, $0 \leq \theta_2 \leq 1$. For $\theta_1 = \theta_2 = 0$, we have the Rotterdam system. For $\theta_1 = 1$ and $\theta_2 = 0$, the CBS system prevails, whereas $\theta_1 = 0$ and $\theta_2 = 1$ result in the AIDS. The coefficients d_i and t_{ij} are related to the coefficients of the original systems by

$$d_i = (1 - \theta_1 - \theta_2)b_i + (\theta_1 + \theta_2)c_1, \tag{50}$$

$$t_{ij} = (1 - \theta_2)s_{ij} + \theta_2 r_{ij}. \tag{51}$$

The properties of these coefficients derive from those for the b_i, c_i, s_{ij}, and r_{ij}. Note that

$$\sum_i d_i = (1 - \theta_1 - \theta_2) \tag{52}$$

and that the negativity property does not hold for t_{ij} with $\theta_2 \neq 0$.

It can be shown that for increasing m, with $0 < 1 - \theta_1 - \theta_2 \leq 1$, the w_i tend to $d_i/(1 - \theta_1 - \theta_2)$, that is, to a constant as in the Rotterdam system. The a_{i0}^M, a_{ik}^M, v_{it}^M satisfy the usual adding up properties. We can express the income elasticities as

$$\eta_i = \frac{d_i}{w_i} + (\theta_1 + \theta_2), \tag{53}$$

and the Slutsky elasticities as

$$\varepsilon_{ij} = \frac{t_{ij}}{w_i} - \theta_2(\delta_{ij} - w_j). \tag{54}$$

It can be verified that with symmetry and homogeneity imposed on the estimation of the t_{ij} the elasticities satisfy all properties implied by demand theory (except negativity for $\theta_2 \neq 0$), even for values of θ_1 and θ_2 outside the [0, 1] interval.

The appeal of (49) is that it can leave somewhat more to be determined by data than would be the case for each of the constituent "elementary" systems while remaining consistent with theory.

13.4 Levels Version

In the preceding section we discussed parametrization in the context of a system in terms of log changes in the system's variables. This context, however, is accidental. What matters is the way in which the elasticities are transformed into constants that satisfy theoretical constraints.

The same approach will be taken to arrive at corresponding levels versions. We will start with a variant of the double-logarithmic demand function (1), namely,

$$\ln q_i = \alpha_i + \eta_i \ln Q' + \sum_j \varepsilon_{ij} \ln p_j, \tag{55}$$

where $\ln Q'$ is a real income variable which we will define later. The ε_{ij} are Slutsky elasticities, which were defined by (8).

Multiplying both sides of (55) by w_i and using Rotterdam specifications (21) and (22), we obtain

$$w_i \ln q_i = a'_{i0} + b_i \ln Q' + \sum_j s_{ij} \ln p_j, \tag{56}$$

with properties (24) through (29) for the b_i and s_{ij}. The a'_{i0} is an additional intercept.

Given adding-up properties (24) and (25), we may write

$$\sum_i w_i \ln q_i = \sum_i a'_{i0} + \ln Q', \tag{57}$$

which in fact defines $\ln Q'$. Using this definition in (56), we can write

$$w_i \ln q_i = (a'_{i0} - b_i \sum_j a'_{j0}) + b_i \sum_j w_j \ln q_j + \sum_j s_{ij} \ln p_j. \tag{58}$$

With

$$a_{i0} = a'_{i0} - b_i \sum_j a'_{j0} \tag{59}$$

and

$$\ln Q = \sum_i w_i \ln q_i, \tag{60}$$

demand equation (58) can be reformulated as

$$w_i \ln q_i = a_{i0} + b_i \ln Q + \sum_j s_{ij} \ln p_j. \tag{61}$$

It follows from (59) that

$$\sum_i a_{i0} = 0. \tag{62}$$

Clearly, $\ln Q$ is a logarithmic quantity index number and thus a natural measure for real income. Its price counterpart is defined as

$$\ln P = \sum_i w_i \ln p_i. \tag{63}$$

We can verify that:

$$\ln m = \ln Q + \ln P - \ln W, \tag{64}$$

with

$$\ln W = \sum_i w_i \ln w_i \tag{65}$$

which contrasts with (18), the corresponding relation for differentials. Here the factor-reversal test is not satisfied. The usual interpretation of real income as deflated nominal income (in this case m/P) does not correspond to this treatment of Q. Note, however, that the (logarithmic) difference $\ln W$ is usually nearly a constant. Its terms, $w_i \ln w_i$, are less variable than w_i, which itself is only variable insofar as preferences are not homothetic. Their sum is at most zero and at least $-\ln n$, with n being the number of goods considered.

However, replacing $\ln Q$ by $\ln(m/P)$ in (61) is not desirable. The adding-up condition will not be satisfied by the full system. There is moreover, no particular reason to prefer $\ln(m/P)$ as the real income indicator over $\ln Q$. The latter has the advantage of being a quantity concept, and thus a real magnitude.

Expression (61) is proposed as the levels version of the Rotterdam system. To it, of course, disturbance terms and eventually other demand determinants are added. To obtain the levels version of the CBS system, we simply replace b_i in (61) by $c_i + w_i$. The result is

$$w_i \ln\left(\frac{q_i}{Q}\right) = a_{i0}^C + c_i \ln Q + \sum_j s_{ij} \ln p_j. \tag{66}$$

On the left-hand side, we have q_i/Q. Here Q can also be seen as a weighted geometric average of the quantities. So q_i/Q is the ratio of q_i to the average of the q_j's. Note that the sum of the variables on the left-hand side equals zero. The intercepts will also add up to zero.

Substituting (40) for s_{ij} in (66) gives the counterpart of (41):

$$w_i(\ln q_i - \ln Q + \ln p_i - \ln P) = a_{i0}^D + c_i \ln Q + \sum_j r_{ij} \ln p_j. \tag{67}$$

The variable on the left-hand side can also be written as

$$w_i(\ln w_i - \ln W) = w_i \ln\left(\frac{w_i}{W}\right). \tag{68}$$

We have here the ratio of w_i to the (weighted geometric) average of the budget shares. Since $w_i/W = (p_i q_i)/(PQ)$, we may also say that it is the ratio of expenditure on i to the (weighted geometric) average of expenditures. Note also that (68) adds up to zero as does the intercept in (67).

Expression (67) can be considered the natural levels counterpart of the AIDS first difference equations. However, the AIDS equations of Deaton and Muellbauer's original proposal are formulated differently. Their levels analogue of (41) is

$$w_i = a_{i0}^A + c_i \ln Q + \sum_j r_{ij} \ln p_j, \tag{69}$$

with the constants a_{i0}^A adding up to one. In fact, as we have already mentioned when presenting the first differences version of AIDS, Deaton and Muellbauer use m/P^* rather than Q, where P^* is either a price index involving the r_{ij} or P as defined by (63). Here the use of Q instead of deflated income is motivated by the desire to have the right-hand side the same as in the other systems. We will therefore consider (69) as our levels version of AIDS. To avoid confusion, equation (67) is taken to represent a separate system, which we will call the W-system.

By construction, the four systems just presented have the same variables on their right-hand sides. Convex combinations of their left-hand sides then also constitute demand systems in levels.

13.5 Alternative Approaches

Another way to derive levels counterparts for the Rotterdam and CBS systems would be to start from AIDS specification (69). Replacing r_{ij} by the right-hand side of (40) and rearranging terms results in an alternative CBS-type equation:

$$w_i\left(1 - \ln\left(\frac{p_i}{P}\right)\right) = a_{i0}^E + c_i \ln Q + \sum_j s_{ij} \ln p_j, \tag{70}$$

where now the variable on the left-hand side and the intercepts add up to one. The dependent variable can also be written as

$$w_i\left(1+\ln\left(\frac{q_i}{Q}\right)-\ln\left(\frac{w_i}{W}\right)\right) \tag{71}$$

which makes it more comparable to the left-hand variable of (66).

Next, replacing the c_i in (70) by $b_i - w_i$ gives an alternative Rotterdam-type equation:

$$w_i\left(1-\ln\left(\frac{p_i}{P}\right)+\ln Q\right)=a_{i0}^F+b_i\ln Q+\sum_j s_{ij}\ln p_j. \tag{72}$$

Its dependent variable is equal to

$$w_i\left(1+\ln q_i-\ln\left(\frac{w_i}{W}\right)\right), \tag{73}$$

which sums up to $1 + \ln Q$. Note that also in this case the intercepts a_{i0}^F add up to one.

The presence of $1 - \ln(w_i/W)$ in both dependent variables (71) and (73) make these alternatives less intuitively plausible. Still, similarity of the right-hand sides of (70) and (72) with those of the systems of the previous section suggests that the class of systems considered there may be extended further. However, we will not discuss them further here.

Another approach to defining cross-sectional demand systems has been explored by Theil (1983). He basically uses the first differences approach. The variables are taken as first differences from one of the observation units. The various systems could be rather easily converted into proper levels versions were it not that $\bar{w}_{ic}$ appears in $\ln Q_c = \sum_i \bar{w}_{ic}(\ln q_{ic} - \ln q_{is})$ and in the dependent variables of the Rotterdam and CBS systems. Here s refers to the observation unit used as the standard from which the differences are taken and c refers to the unit described. Thus $\bar{w}_{ic} = (w_{ic} + w_{is})/2$. We cannot simply write the differences as differences between two terms of which one is constant across c. The estimation results will generally depend on the observation unit used as standard. It is not clear which unit should be taken as standard.

In such a system we could of course replace $\Delta\ln Q_c$ by $\ln Q_c - \ln Q_s$ with $\ln Q_c = \sum_i w_{ic}\ln q_{ic}$ as in (60). Likewise, we could replace the left-hand side variable of the Rotterdam system by $w_{ic}\ln q_{ic} - w_{is}\ln q_{is}$ and that of the CBS system by $w_{ic}\ln(q_{ic}/Q_c) - w_{is}\ln(q_{is}/Q_s)$. Reordering of terms brings us back to the levels version of the previous section with somewhat more elaborate intercepts. The results will not depend on the unit used as standard. In fact there is no need to single out any unit for that purpose.

If we can use the original Theil approach to choose one unit as the standard one, we have another set of three demand systems with mutually related parametrizations. Their right-hand sides contain the same variables. However, these are not the same as those of the level versions of section 13.4.

13.6 Simulation

By estimating demand systems, we obtain information about coefficients, elasticities, or partial derivatives. The final use of demand systems is to provide information about the quantities demanded. Given m and the prices, knowledge of the budget shares is equivalent. The left-hand sides of the Rotterdam and CBS specifications are not simple functions of the quantities and/or budget shares. Their derivation from the calculated values of the right-hand sides deserves some discussion in view of the possibility of comparing their ability to correctly simulate the actual quantities or budget shares. In this context simulation does not refer to the use of artificial random data generation processes.

The case of AIDS seems to create few problems. The left-hand variable of its levels version is the budget share itself. Assuming that the prices are exogenously given, there are two possible simulations: with Q given and m not, and the reverse of this. If Q is given, the w_i are easily calculated for given values of the coefficients in (69). To solve for Q, however, we need m. This variable is endogenously determined by (64) by using the calculated w_i as weights in (63) and (65). To apply (69) when m rather than Q is given, we proceed first by calculating Q, for which (64) can also be employed. Now, the w_i needed for (63) and (65) are not available. An iterative solution procedure is needed, starting from provisional values for w_i, calculating $\ln Q$, applying (69) to obtain new values for w_i, which serve as the starting values for the new round. This sequence is repeated until successive changes in w_i become smaller in absolute value than some specified minimum. Usually a few iterations are sufficient for this purpose.

Simulation of budget shares and quantities demanded for AIDS in first differences does not create any additional problems. Note that there is no guarantee that simulated w_i stay within the interval between zero and one.

They will add up to one, but some w_i may be negative and others larger than unity.

Simulating with Rotterdam system (61) requires further treatment of the left-hand side variable $w_i \ln q_i$. We will first transform it into an expression in w_i:

$$w_i \ln q_i = w_i(\ln w_i - \ln p_i + \ln m) = w_i(\ln w_i + z_i), \tag{74}$$

with $z_i = \ln m - \ln p_i$. Let y_i be the calculated value of the right-hand side of (61). We then look for the w_i that solves

$$w_i(\ln w_i + z_i) = y_i. \tag{75}$$

Such a solution might not exist. The left-hand side reaches for $w_i = \exp(-1 - z_i)$, its minimum of $-\exp(-1 - z_i)$. If y_i is less than this value, there is no solution to (75). If y_i is larger, there are two solutions: one larger than $\exp(-1 - z_i)$, and the other smaller. The latter will always be non-negative; the larger may be greater than unity and hence be inadmissible. If there are two admissible solutions, a choice has to be made. Often one of the two solutions is rather improbable, leaving one acceptable solution. This, however, cannot be guaranteed in general.

A further aspect of simulation with (61) is similar to the one discussed for AIDS. If Q is given rather than m, there is no problem in obtaining y_i, but we need to calculate m to arrive at z_i. Therefore an iterative procedure is needed. If m is given rather than Q, the reverse happens: z_i can be readily found, but to obtain y_i, we need to calculate Q first, for which w_i are needed. Here also an iterative solution procedure has to be used.

From the point of view of simulation, Rotterdam variant (72) has a simpler left-hand side variable. There are no multiple solutions. Iteration is needed to determine P and, if m is given, to determine Q. Solutions for w_i outside the 0–1 interval may occur.

The possibility of no solution or a two-valued solution also arises in the case of the Rotterdam system in first differences. The situation is slightly different from that of (75) because the equation to be solved is

$$\bar{w}_{it}(\ln w_{it} + z_{it}) = y_{it}, \tag{76}$$

with $\bar{w}_{it}$ on the left-hand side. Here $z_{it} = \ln m_t - \ln p_{it} - \ln q_{i,t-1}$, whereas y_{it} is calculated from the right-hand side of (32) by setting $v_{it} = 0$. The simulation can be made more straightforward when in (76) if $\bar{w}_{it}$ is replaced by predetermined $w_{i,t-1}$.

Similar problems and possibilities exist for the various versions of the CBS system.

From the discussion of determining m if Q is given, it is clear that m is the expenditure needed to pay for the optimal bundle given Q. It is the left-hand side variable of the expenditure function. By simulating with varying prices and constant Q, we can numerically generate price index numbers as the ratios of the m's needed to obtain $\ln Q$ in the two price systems.

13.7 Comparing Empirical Performance

Demand systems are tools for the empirical analysis of consumer behavior. To compare their empirical performance seems natural. However, it is not possible to draw general conclusions from the results for a particular sample or a set of samples. Still, some experimentation can be informative.

Our experiments will involve only the levels versions (61), (66), and (69) of the Rotterdam system, the CBS system, and AIDS, respectively. The comparison should shed some light on the relative merits of the particular parametrizations. The matrix of price coefficients s_{ij} of the Rotterdam and CBS systems will be estimated without imposing negativity condition (28) to maintain comparability with AIDS where such a condition cannot be implemented.

The Data

The three systems are estimated for a cross section of 34 countries in 1975. The U.N. International Comparison Project (ICP) has collected price and quantity data for 151 categories of consumer demand, which have been published by Kravis, Heston, and Summers (1982). The countries of the sample range from (poor, e.g., Malawi) to rich (e.g., the United States). Their price systems show considerable variation. These data seem well suited to tests of empirical performance.

For our purpose the 151 categories of consumer demand are more than is necessary. We have aggregated them into eight major groups. One of these is food. Its budget share ranges from 68 percent for Sri Lanka to 16 percent for the United States—an indication of the wide range of variability in this data set.

It is obvious that the differences in demand behavior across this set of countries have to be attributed to more than differences in prosperity and price structure. One source of difference is that of climate; another is that of the age composition of the population. Altogether six additional variables, taken from Barten and Summers (1986), have been used to account for other determinants of demand than average income and prices. They are mean annual temperature, the average temperature of the coldest month, the average temperature of the warmest month, the percentage of children under 15 years of age, the Gini index of inequality of the income distribution, and the logarithm of the population size. Note that the ICP is already expressed in per capita terms. Population size as an additional variable includes possible economies of scale. These six variables are selected from a class of twelve. The desire not to waste degrees of freedom limited their number to six.

There are many reasons why any demand system would be inadequate to describe the variation in behavior across countries. Demand systems reflect characteristics of individual consumer demand, whereas the data refer to countries in the aggregate. In spite of the enormous effort of the ICP to arrive at comparable data, there is still much disparity. The additional variables are perhaps also not representative enough to absorb explainable variation across countries. The omitted variables could be correlated with the income and price variables causing biases in coefficient estimators. More reasons for the inadequacy in describing behavior can be advanced. Still it is interesting to find out the extent to which the data agree with the proposed models.

The Coefficients of Determination

The DEMMOD computer program has been used to estimate systems of demand equations by maximum likelihood procedures as described in Barten (1969) and Barten and Geyskens (1975). This program calculates the R^2 for each commodity group. But all equations are estimated jointly, and the R^2's are not maximized. Still, they may serve as a simple measure of relative fit (see table 13.1). A simple inspection of the R^2's in table 13.1 shows that CBS scores best, followed by Rotterdam, with AIDS being the weakest.

Information Inaccuracy

One could argue that the R^2's are not really comparable across the three systems because the left-hand side variables differ. One way to test comparability is to let the estimated version of each system generate budget

Table 13.1
Coefficients of determination (R^2)

Commodity group	Rotterdam system	CBS system	AIDS
1. Food	0.811	0.829	0.892
2. Clothing and footwear	0.726	0.910	0.656
3. Housing and fuel	0.699	0.723	0.575
4. Household furnishings and operations	0.788	0.890	0.698
5. Medical care	0.904	0.892	0.890
6. Transport and communications	0.824	0.898	0.689
7. Education	0.507	0.748	0.566
8. Remainder	0.809	0.789	0.687

Table 13.2
Average information accuracy

System	Full sample	Reduced sample
Rotterdam	0.0326	0.0326
CBS	0.1785	0.1172
AIDS	0.0141	0.0141

shares (see the preceding section) and then to compare the simulated budget shares with the actual ones.

A useful aggregate measure of the divergence between observed and predicted budget shares is Theil's (1967) concept of information inaccuracy, which is defined as

$$I_c = \sum_i w_{ic}(\ln w_{ic} - \ln \hat{w}_{ic}), \tag{77}$$

where w_{ic} refers to the observed and $\hat{w}_{ic}$ to the calculated budget shares for commodity i and country c. For our purpose we will use the average information inaccuracy:

$$I = \frac{\sum_c I_c}{34}. \tag{78}$$

The results are given in table 13.2.

A lower value for I means better performance. From table 13.2 it is clear that AIDS dominates the other two systems. The CBS system is particularly weak. Calculating predicted shares for AIDS did not cause any problems.

For the Rotterdam and CBS systems, the work was less simple. The simulation failed to converge for at least one country using the Rotterdam system and for no less than thirteen countries using the CBS system. Omitting these countries from the calculation of I gives the results of the last column of table 13.2. The picture has not changed drastically.

It is not clear whether this remaining divergence in predictive behavior is due to shortcomings in the simulation procedure, or to the fact that predicting w_{ic} is just what AIDS is optimizing, or even to the superiority of the AIDS parameterization for this type of data.

Income and Price Elasticities

Another way to compare the three systems is to evaluate the implied income and price elasticities to see to what extent they correspond to theoretical and intuitive prior ideas. For all three system, the elasticities are not estimated as such but they can be calculated from the estimated coefficients and the budget shares for a particular country. In this case the elasticities are evaluated for Italy because its budget shares correspond closely to the average elasticities for the whole sample.

In table 13.3 are listed the values of the elasticity of demand for a commodity with respect to Q, the "income" elasticity, and in table 13.4 the

Table 13.3
Income elasticities for Italy

Commodity group	Rotterdam system	CBS system	AIDS
1. Food	0.38	0.81	0.86
	(0.59)	(0.11)	(0.09)
2. Clothing and footwear	0.85	1.10	0.99
	(0.71)	(0.06)	(0.11)
3. Housing and fuel	2.46	1.27	1.24
	(0.92)	(0.14)	(0.14)
4. Household furnishings and operations	1.97	0.96	1.18
	(1.03)	(0.12)	(0.17)
5. Medical Care	1.35	1.04	1.12
	(0.60)	(0.08)	(0.08)
6. Transport and communications	1.85	1.21	1.13
	(0.70)	(0.08)	(0.12)
7. Education	−0.63	0.74	0.71
	(1.06)	(0.17)	(0.16)
8. Remainder	0.81	1.14	1.02
	(0.88)	(0.13)	(0.13)

values of the Slutsky elasticity of demand for a commodity with respect to its own price. In parentheses under the elasticity values are the standard errors.

Form table 13.3 it appears that the income elasticities for CBS and AIDS are rather similar, as one would expect of the same type of parametrization for the effect of real income. Also the standard errors are roughly equal. All of the elasticities turn out to be close to unity. This reflects the fact that the underlying c_i are close to zero.

As indicated in section 13.3, the nonzero c_i cause problems for asymptotic behavior. With zero c_i, such problems are avoided. The present sample with its wide variation in Q ($Q_{max}/Q_{min} = 12.6$) seems to force the c_i toward zero.

Zero c_i suggest linear Engel curves. The Rotterdam system should agree with that. The results of table 13.3 are not in accordance with this expectation. There is a substantial and unusual variation in the Rotterdam income elasticities, which is suspicious. Moreover the standard errors are fairly large. The Rotterdam specification does not seem to adjust very gracefully to the wide variation in Q and in the budget share of this particular sample.

The inadequate performance of the Rotterdam system reveals itself also in the estimated values of the own Slutsky elasticities. Only three out of

Table 13.4
Own Slutsky elasticities for Italy

Commodity group	Rotterdam system	CBS system	AIDS
1. Food	2.67	−0.19	−0.18
	(0.95)	(0.09)	(0.15)
2. Clothing and footwear	−2.04	−0.93	−1.06
	(0.92)	(0.08)	(0.07)
3. Housing and fuel	1.65	−0.38	−0.51
	(0.87)	(0.12)	(0.13)
4. Household furnishings and operations	−2.14	−1.07	−1.19
	(1.25)	(0.17)	(0.21)
5. Medical Care	2.44	−0.34	−0.44
	(0.56)	(0.08)	(0.07)
6. Transport and communications	1.99	−0.62	−0.46
	(0.91)	(0.11)	(0.07)
7. Education	0.49	−0.78	−0.70
	(0.77)	(0.13)	(0.11)
8. Remainder	−2.31	−1.00	−1.03
	(1.43)	(0.23)	(0.17)

the eight elasticities have the theoretically expected negative sign. The two other systems have no problem with the negativity of the own Slutsky elasticity. Here also the Rotterdam elasticities are large in absolute value, like the corresponding standard errors.

Despite the different parametrizations of price coefficients for CBS and AIDS, the implied elasticity values for Italy are rather close. They are roughly comparable in size to what one usually obtains for such elasticities for highly aggregate commodity groups with few, if any, close substitutes.

The difference between the Rotterdam and the CBS Slutsky elasticities is then even more surprising since they are based on the same parametrization. The difference in the specification of the effect of real income appears to be dominating.

A More Formal Test

The comparisons discussed so far have been descriptive. It is not easy to assess the statistical significance of differences in performances. Note that the systems considered are not nested. The well-established theory of model selection when the various alternatives are nested cannot be applied.

The present set of systems distinguishes itself from the usual context of nonnested model selection by having the same right-hand sides. This property can be conveniently exploited.

Consider, for example, the following linear combination of the Rotterdam and CBS dependent variables:

$$(1 - \theta)w_{ic} \ln q_{ic} + \theta w_{ic} \ln\left(\frac{q_{ic}}{Q_c}\right). \tag{79}$$

For a given value of θ, we can estimate the coefficients on the right-hand side in the usual way and obtain a (maximum) likelihood value. Clearly, for $\theta = 0$, we have the maximum likelihood value for the Rotterdam system, and for $\theta = 1$, the maximum likelihood value for the CBS system.

We can, of course, also estimate θ itself by maximum likelihood procedures. Under either hypothesis it will be a consistent estimator of 0 or 1, respectively. The greater proximity to one of these values in finite samples is then seen as a rejection of the empirical validity of the other.

In the present case θ was estimated by maximizing the likelihood function concentrated on θ only. The square root of the reciprocal of the second-order derivative of the quadratic approximation evaluated at the maximizing value for θ serves as its standard error.

Table 13.5
Logarithmic likelihood values and test statistics

Rotterdam/CBS		Rotterdam/AIDS		CBS/AIDS	
θ	ln L	θ	ln L	θ	ln L
0	245	0	245	0	742
1	742	1	683	1	683
1.14	818	1.18	1062	0.21	751
(0.01)		(0.00)		(0.01)	

The same approach can be used for the pairwise comparison between the Rotterdam system and AIDS and between the CBS system and AIDS. Note that this test is symmetric for the two alternatives in each pair—that is, replacing θ by $1 - \xi$ in (79) will simply reverse the roles of the two alternatives, but the optimizing ξ will be one minus the optimizing θ.

The results for the optimizing θ values are given in table 13.5, together with (in parentheses) their standard errors and the corresponding logarithmic likelihood values. To complete the picture, the logarithmic likelihood values for the elementary systems are given as well.

From the table 13.5 it is obvious that the Rotterdam system is dominated by both the CBS system and AIDS. The optimizing θ values are in both comparisons closer to one than to zero. The small standard errors reflect the sharp peak in the likelihood function at the relevant point. (They may overstate the small sample precision of the optimizing θ values.) The 0.21 value for θ in the CBS/AIDS comparison may be interpreted as a rejection of AIDS.

We might argue that the substantial increases in the likelihoods when θ is estimated suggests the rejection of all three systems in favor of some hybrid system. It might very well be that each system is too rigid in its parametrization and that some simple relaxation may improve the empirical performance drastically. It is beyond the scope of this chapter to investigate this approach further.

A Final Evaluation

The various comparisons of the three systems have not created an entirely clear picture. The Rotterdam system appears to be the least satisfactory as far as coefficients estimates and likelihood value are concerned. The CBS system does not fare well in simulation. The R^2's of AIDS are relatively weak. Still, apart from the simulation problem, CBS seems to be best. AIDS is a close runner-up. The specification of the effect of real income appears to be crucial.

As mentioned at the beginning of this section, these conclusions are specific for the data set used and do not necessarily carry over to an application of levels versions to, for instance, time series data.

13.8 Concluding Remarks

We started with specifications for the Rotterdam, CBS, and almost ideal demand systems for first differences in the logarithm of the relevant variables and derived analogous systems for the levels of the logarithms of those variables. In each system the right-hand side was the same but not the left-hand side. Nevertheless, even with the same coefficients, more than one variant could be used on the left-hand sides.

The Rotterdam parametrization uses constant marginal budget shares and constant price coefficients that are simple transformations of the Slutsky elasticities and therefore easy to interpret. The price coefficients of AIDS are less convenient in that respect. AIDS takes the difference between the marginal and average budget shares as constant. The CBS system uses the same type of income coefficients as AIDS and the Rotterdam type of price coefficients.

The constant marginal budget shares used in the Rotterdam system imply constant average shares for high budget levels. Similarly, keeping the difference between the marginal and average budget shares constant (AIDS and CBS) is only possible for high budget levels if this difference is zero. Equal marginal and average budget shares means that both are constant. In this respect the three systems are less different than appears on first sight.

The 1975 ICP cross section of 34 countries displays considerable variation in the variables. It has a wide (real) income range and widely varying budget shares. Because the variations in budget shares cannot be attributed to differences in price structures and determinants other than (real) income, one might expect all three systems not to describe these data well. But it appears that the CBS specification has hardly any problem with this. Its implied values for income elasticities are close to one, however, suggesting independence of the budget shares from income. For AIDS a similar conclusion can be made. The Rotterdam specification seems to agree less well with these data. This is no doubt due to the use of constant marginal budget shares.

The development of the various systems has also introduced hybrid forms—linear (convex) combinations of the underlying elementary systems. These might offer better adjustment to the data, although somewhat less suitable interpretation of the results. However, as our experiments indicate, there is still room for improvement of the empirical performance of the various elementary systems.

Note

The author is indebted to Leon Bettendorf for his assistance on the empirical applications. He also thanks an anonymous referee for his constructive remarks. The author remains solely responsible for possible errors. Research for this project was supported by the Belgian Science Foundation (FKFO) and the Research Fund of the Katholieke Universiteit Leuven.

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